

A resolution of the inclusive flavor-breaking τ $|V_{us}|$ puzzle

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Abstract

We revisit the puzzle of $|V_{us}|$ values obtained from the conventional implementation of hadronic- τ -decay-based flavor-breaking finite-energy sum rules lying $> 3\sigma$ below the expectations of three-family unitarity. Significant unphysical dependences of $|V_{us}|$ on the choice of weight, w , and upper limit, s_0 , of the experimental spectral integrals entering the analysis are confirmed, and a breakdown of assumptions made in estimating higher dimension, $D > 4$, OPE contributions is identified as the main source of these problems. A combination of continuum and lattice results is shown to suggest a new implementation of the flavor-breaking sum rule approach in which not only $|V_{us}|$, but also $D > 4$ effective condensates, are fit to data. Lattice results are also used to clarify how to reliably treat the slowly converging $D = 2$ OPE series. The new sum rule implementation is shown to cure the problems of the unphysical w - and s_0 -dependence of $|V_{us}|$ and to produce results ~ 0.0020 higher than those of the conventional implementation. With B-factory input, including a new preliminary result for the exclusive $\tau \rightarrow K^- \pi^0 \nu_\tau$ branching fraction, we find $|V_{us}| = 0.2229(22)_{exp(4)_{th}}$, in excellent agreement with the result from $K_{\ell 3}$, and compatible within errors with the expectations of three-family unitarity, thus resolving the long-standing inclusive τ $|V_{us}|$ puzzle.

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I. INTRODUCTION

With $|V_{ud}| = 0.97417(21)$ [1] as input and $|V_{ub}|$ negligible, 3-family unitary implies $|V_{us}| = 0.2258(9)$. Direct determinations of $|V_{us}|$ from $K_{\ell 3}$ and $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, using recent 2014 FlaviaNet experimental results [2] and 2016 lattice input [3] for $f_+(0)$ and f_K/f_π , respectively, yield results, $|V_{us}| = 0.2231(9)$ and $0.2253(7)$, in agreement with this expectation. In contrast, the most recent update [4] of the conventional implementation of the finite-energy sum rule (FESR) determination employing flavor-breaking (FB) combinations of inclusive strange and non-strange hadronic τ decay data [5], yields $|V_{us}| = 0.2176(21)$, 3.6σ below 3-family-unitarity expectations. The general FB FESR framework whose conventional implementation produces this result is outlined below.

In the Standard Model, the differential distributions, $dR_{V/A;ij}/ds$, for flavor $ij = ud, us$, vector (V) or axial-vector (A) current-mediated decays, with $R_{V/A;ij}$ defined by $R_{V/A;ij} \equiv \Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons}_{V/A;ij}(\gamma)]/\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]$, are related to the spectral functions, $\rho_{V/A;ij}^{(J)}$, of the $J = 0, 1$ scalar polarizations, $\Pi_{V/A;ij}^{(J)}$, of the corresponding current-current two-point functions, by [6]

$$\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[w_\tau(y_\tau) \rho_{V/A;ij}^{(0+1)}(s) - w_L(y_\tau) \rho_{V/A;ij}^{(0)}(s) \right], \quad (1)$$

where $y_\tau = s/m_\tau^2$, $w_\tau(y) = (1-y)^2(1+2y)$, $w_L(y) = 2y(1-y)^2$, S_{EW} is a known short-distance electroweak correction [7], and V_{ij} is the flavor ij CKM matrix element. The $J = 0$ spectral functions, $\rho_{A;ud,us}^{(0)}(s)$, are dominated by the accurately known, chirally unsuppressed π and K pole contributions. The remaining, continuum contributions to $\rho_{V/A;ud,us}^{(0)}(s)$ are $\propto (m_i \mp m_j)^2$, and hence negligible for $ij = ud$. For $ij = us$, they are small (though not totally negligible) and highly constrained, through the associated $ij = us$ scalar and pseudoscalar sum rules, by the known value of m_s , making possible mildly model-dependent determinations in the range $s \leq m_\tau^2$ relevant to hadronic τ decays [8, 9]. Subtracting the resulting $J = 0$ contributions from the RHS of Eq. (1) yields the $J = 0 + 1$ analogue, $dR_{V/A;ij}^{(0+1)}/ds$, of $dR_{V/A;ij}/ds$, from which the $J = 0 + 1$ spectral function combinations $\rho_{V/A;ud,us}^{(0+1)}(s)$ can be determined.

The inclusive τ determination of $|V_{us}|$ employs FB FESRs for the spectral function combination, $\Delta\rho \equiv \rho_{V+A;ud}^{(0+1)} - \rho_{V+A;us}^{(0+1)}$ and associated polarization difference, $\Delta\Pi \equiv \Pi_{V+A;ud}^{(0+1)} - \Pi_{V+A;us}^{(0+1)}$ [5], generically, for any $s_0 > 0$ and any choice of analytic weight $w(s)$,

$$\int_0^{s_0} w(s) \Delta\rho(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s) \Delta\Pi(s) ds. \quad (2)$$

For large enough s_0 , the OPE is used on the RHS.

Defining the re-weighted integrals

$$R_{V+A;ij}^w(s_0) \equiv \int_0^{s_0} ds \frac{w(s)}{w_\tau(s)} \frac{dR_{V+A;ij}^{(0+1)}(s)}{ds}, \quad (3)$$

and using Eq. (2) to replace the FB difference

$$\delta R_{V+A}^w(s_0) \equiv \frac{R_{V+A;ud}^w(s_0)}{|V_{ud}|^2} - \frac{R_{V+A;us}^w(s_0)}{|V_{us}|^2}, \quad (4)$$

with its OPE representation, one finds, solving for $|V_{us}|$ [5],

$$|V_{us}| = \sqrt{R_{V+A;us}^w(s_0) / \left[\frac{R_{V+A;ud}^w(s_0)}{|V_{ud}|^2} - \delta R_{V+A}^{w,OPE}(s_0) \right]}. \quad (5)$$

The result is necessarily independent of s_0 and w so long as all input is reliable. Assumptions employed in evaluating $\delta R_{V+A}^{w,OPE}(s_0)$ can thus be tested for self-consistency by varying w and s_0 . OPE assumptions entering the conventional implementation of the FB FESR approach in fact produce $|V_{us}|$ displaying significant w - and s_0 -dependence [24].

The $> 3\sigma$ low $|V_{us}|$ results noted above are produced by a conventional implementation of the general FB FESR framework, Eq. (5), in which a single s_0 ($s_0 = m_\tau^2$) and single weight ($w = w_\tau$), are employed [5]. This restriction allows the $ij = ud$ and us spectral integrals to be determined from the inclusive ud and us branching fractions alone, but precludes carrying out s_0 - and w -independence tests. Since w_τ has degree 3, $\delta R_{V+A}^{w_\tau,OPE}(s_0)$ receives contributions up to dimension $D = 8$. While $D = 2$ and 4 contributions, determined by α_s and the quark masses and condensates [3, 20–23], are known, $D > 4$ contributions are not. In the conventional implementation, $D = 6$ contributions are estimated using the vacuum saturation approximation (VSA) and $D = 8$ contributions neglected [5, 24]. These assumptions are potentially dangerous since the FB V+A VSA estimate involves a very strong double cancellation¹, and the VSA is known to be badly violated, in a channel-dependent manner, from studies in the non-strange sector [25].

Such assumptions can, in principle, be tested by varying s_0 . Writing $D > 4$ contributions to $\Delta\Pi(Q^2)$ as $\sum_{D>4} C_D/Q^D$, with C_D the effective dimension D condensate, the integrated $D = 2k + 2$ OPE contribution to the RHS of Eq. (2), for a polynomial weight $w(y) = \sum_{k=0} w_k y^k$ with $y = s/s_0$, is, up to α_s -suppressed logarithmic corrections,

$$- \frac{1}{2\pi i} \oint_{|s|=s_0} ds w(y) [\Delta\Pi(s)]_{D=2k+2}^{OPE} = (-1)^k w_k \frac{C_{2k+2}}{s_0^k}. \quad (6)$$

Problems with the assumptions employed for C_6 and C_8 in the conventional implementation will thus manifest themselves as an unphysical s_0 -dependence in the $|V_{us}|$ results obtained using weights $w(y)$ with non-zero coefficients, w_2 and/or w_3 , of y^2 and y^3 .

Another potential issue for the FB FESR approach is the slow convergence of the $D = 2$ OPE series. To four loops, neglecting $O(m_{u,d}^2/m_s^2)$ corrections [20]

$$[\Delta\Pi(Q^2)]_{D=2}^{OPE} = \frac{3}{2\pi^2} \frac{\bar{m}_s}{Q^2} \left[1 + \frac{7}{3} \bar{a} + 19.93 \bar{a}^2 + 208.75 \bar{a}^3 \right], \quad (7)$$

¹ A factor of ~ 3 reduction occurs when the individual ud and us V+A sums are formed, and a further factor of ~ 6 reduction when the FB $ud - us$ V+A difference is formed.

where $\bar{a} = \alpha_s(Q^2)/\pi$, and $\bar{m}_s = m_s(Q^2)$, $\alpha_s(Q^2)$ are the running strange mass and coupling in the \overline{MS} scheme. With $\bar{a}(m_\tau^2) \simeq 0.1$, the ratio of $O(\bar{a}^3)$ to $O(\bar{a}^2)$ terms is > 1 at the spacelike point on $|s| = s_0$ for all kinematically accessible s_0 . Such slow “convergence” complicates choosing an appropriate truncation order and estimating the associated truncation uncertainty.

No apparent convergence problem exists for the $D = 4$ series, which, to three loops, dropping numerically tiny $O(m_q^4)$ terms, is given by [21]

$$[\Delta\Pi(Q^2)]_{D=4}^{OPE} = \frac{2 [\langle m_u \bar{u}u \rangle - \langle m_s \bar{s}s \rangle]}{Q^4} \left(1 - \bar{a} - \frac{13}{3}\bar{a}^2 \right). \quad (8)$$

The slow convergence of the $D = 2$ OPE series and the reliability of conventional implementation assumptions for C_6 and C_8 will be investigated in the next section.

In the rest of the paper, the non-strange and strange spectral distributions entering the various FESRs considered are fixed using $\pi_{\mu 2}$, $K_{\mu 2}$ and SM expectations for the π and K pole contributions, recent ALEPH data for the continuum ud V+A distribution [10], Belle [11] and BaBar [12, 13] results for the $K^-\pi^0$ and $\bar{K}^0\pi^-$ distributions, BaBar results [14] for the $K^-\pi^+\pi^-$ distribution, Belle results [15] for the $\bar{K}^0\pi^-\pi^0$ distribution, a combination of BaBar [17] and Belle [18] results for the very small $\bar{K}\bar{K}K$ distribution, and 1999 ALEPH results [16] for the combined distribution of those strange modes not remeasured by the B-factory experiments. The overall scales for the BaBar and Belle exclusive strange mode distributions have been set, with one exception, by scaling the unit-normalized experimental distributions with the corresponding 2014 HFAG [19] exclusive branching fractions. The exception is the $K^-\pi^0$ mode, where two choices exist for $B[\tau^- \rightarrow K^-\pi^0\nu_\tau]$: the 2014 HFAG summer fit result 0.00433(15) [19], and the preliminary BaBar thesis result 0.00500(14) [13]. Since the latter is favored by BaBar, whose earlier result dominates the 2014 HFAG average, our central values below are obtained using this choice. Finally, with the total strange branching fraction differing slightly from that used by ALEPH in setting its overall ud V+A normalization, a very small rescaling is applied to the ALEPH continuum ud V+A distribution to restore the sum of the electron, muon, non-strange and strange branching fractions to unity.

II. TESTING CONVENTIONAL IMPLEMENTATION ASSUMPTIONS

The conventional implementation assumptions, $C_6 \simeq C_6^{VSA}$ and $C_8 = 0$, can be efficiently investigated using appropriately chosen s_0 - and w -independence tests. A comparison of the results of the $w_\tau(y) = 1 - 3y^2 + 2y^3$ and $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$ FESRs is particularly illuminating since the coefficients of y^2 in the two weights differ only by a sign. The corresponding integrated $D = 6$ OPE contributions are thus identical in magnitude but opposite in sign. If, as the VSA estimate suggests, $D = 6$ contributions are small for w_τ , they must also be small for \hat{w} . Similarly, if integrated $D = 8$ contributions are negligible for w_τ , those for \hat{w} , which are $-1/2$ times as large, will also be negligible. If conventional implementation assumptions for C_6 and C_8 are reliable, the $|V_{us}|$ obtained from the w_τ and \hat{w} FESRs should thus be in good agreement, and show

good individual s_0 stability. In contrast, if these assumptions are not reliable, and $D = 6$ and $D = 8$ contributions are not both small, one should see s_0 -instabilities of opposite signs in the two cases, and s_0 -dependent differences in the results from the two FESRs which decrease with increasing s_0 . The results of this comparison, shown in the left panel of Figure 1, clearly correspond to the second scenario.

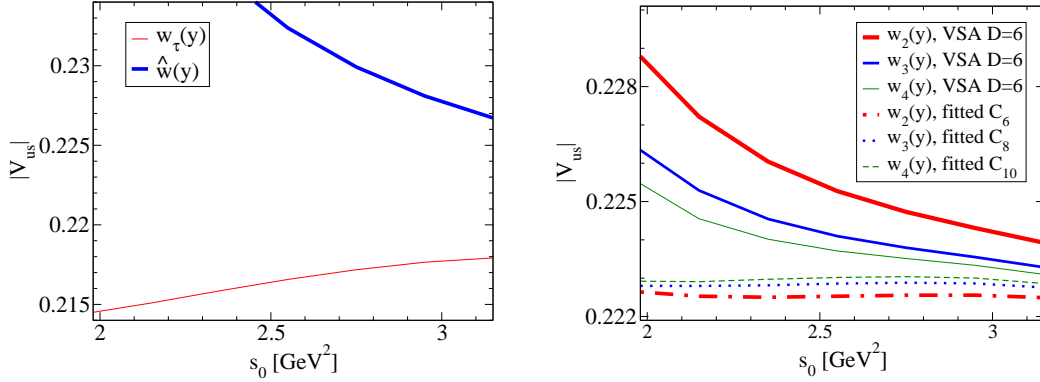


FIG. 1: Left panel: conventional implementation w_τ (top curve) and \hat{w} (bottom curve) results for $|V_{us}|$. Right panel: w_N FESR results obtained using the fixed-order (FOPT) $D = 2$ prescription (solid lines, top to bottom: conventional implementation w_2 , w_3 , and w_4 results; dashed lines, bottom to top: w_2 , w_3 and w_4 results obtained using central $C_{6,8,10}$ fit values from the alternative FB FESR analyses described in the text.

The right panel of Fig. 1 shows the results of additional w - and s_0 -independence tests involving the weights $w_N(y)$, $N = 2, 3, 4$, with²

$$w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N. \quad (9)$$

The solid lines show the results obtained using the conventional implementation treatment of $D > 4$ OPE contributions, the dashed lines those produced by the alternate implementation discussed below, in which the $D > 4$ effective condensates are fit to experimental data. The s_0 -dependent, conventional implementation results for all of w_τ , \hat{w} , w_2 , w_3 and w_4 show evidence of converging toward a common value at $s_0 > m_\tau^2$, as expected if the observed s_0 -instabilities result from $D > 4$ OPE contributions larger than those taken as input in the conventional implementation.

The impact of the slow convergence of the $D = 2$ OPE series can be investigated by comparing OPE expectations to lattice results for $\Delta\Pi(Q^2)$ over a range of Euclidean

² The $w_N(y)$, like $w_\tau(y)$, have a double zero at $s = s_0$ ($y = 1$). This serves to keep duality violating (DV) contributions safely small above $s \simeq 2 \text{ GeV}^2$ [26].

$Q^2 = -q^2 = -s$, using variously truncated versions of the $D = 2$ OPE series. Lattice results were obtained using the RBC/UKQCD $n_f = 2 + 1$, $32^3 \times 64$, $1/a = 2.38$ GeV, domain wall fermion ensemble with $m_\pi \sim 300$ MeV [27]. A tight cylinder cut, with a radius determined in a recent study of the extraction of α_s from lattice current-current two-point function data [28], was imposed to suppress lattice artifacts at higher Q^2 . The values of the light quark masses, $m_u = m_d \equiv m_\ell$ and m_s , for this ensemble, determined in Ref. [27], were used for determining the corresponding OPE expectations.

We consider the comparison first for larger Q^2 , where $D = 2$ and 4 contributions should dominate. The $D = 2$ OPE contribution is determined using ensemble values of m_u and m_s [27], the central PDG value for α_s [22], and considering 2-, 3- and 4-loop truncation of the $D = 2$ series. Both fixed scale, $\mu^2 = 4 \text{ GeV}^2$, and local scale, $\mu^2 = Q^2$, choices for handling the logarithms in the truncated series are considered. The former choice is the analogue of the fixed order (FOPT) prescription for the FESR contour integrals, the latter the analogue of the alternate contour-improved (CIPT) prescription.³ For $D = 4$ contributions, Eq. (8), we employ the Gell-Mann-Oakes-Renner relation for $\langle m_u \bar{u}u \rangle$ and fix $\langle m_s \bar{s}s \rangle$ using the ensemble value of m_s/m_ℓ , translating the HPQCD result for $\langle \bar{s}s \rangle / \langle \bar{\ell}\ell \rangle$ at physical quark masses [23], to that for the ensemble masses using NLO ChPT [29].

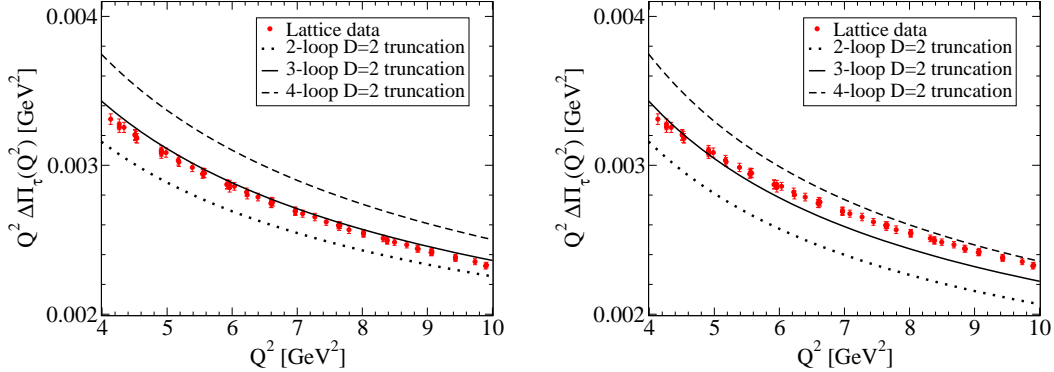


FIG. 2: Comparison of lattice results and $D = 2 + 4$ OPE expectations for $Q^2 \Delta \Pi_\tau(Q^2)$, for either fixed-scale (left panel) or local-scale (right panel) treatments of the $D = 2$ series.

The comparisons obtained using the fixed- and local-scale versions of the $D = 2$ series are shown in the left and right panels of Fig. 2, respectively. The best representation of the lattice results is provided by the 3-loop-truncated, fixed-scale version, which produces an excellent match over a wide range of Q^2 , extending from near $\sim 10 \text{ GeV}^2$ down to just above $\sim 4 \text{ GeV}^2$. The Q^2 dependence of the lattice results clearly favors the fixed-scale over the alternate local-scale treatment.

³ The FOPT prescription evaluates weighted $D = 2$ OPE integrals using a fixed scale choice (usually $\mu^2 = s_0$), the CIPT prescription using the local-scale choice, $\mu^2 = Q^2$.

Comparison to the lattice results also provides two further useful pieces of information. The left panel of Fig. 3 shows the comparison of the lattice results, the three-loop-truncated, fixed-scale $D = 2$ series version of the $D = 2 + 4$ OPE sum, and this same $D = 2 + 4$ OPE sum now supplemented by the VSA estimate for $D = 6$ contributions, in the lower Q^2 region. Below $\sim 4 \text{ GeV}^2$, the lattice results clearly require $D > 4$ OPE contributions much larger than those assumed in the conventional implementation, confirming the conclusions reached already from the $w_\tau\hat{w}$ FESR comparison above. The right panel shows the comparison of the lattice results and three-loop-truncated, fixed-scale $D = 2$ series $D = 2 + 4$ OPE sum, now with the conventionally estimated errors for the latter also displayed. These are obtained by combining in quadrature standard estimates for the $D = 4$ truncation errors with uncertainties produced by those on the input $D = 2$ and 4 OPE parameters. Despite the apparently problematic convergence behavior of the $D = 2$ series, conventional OPE error estimates are seen to provide an extremely conservative assessment of the uncertainty for the $D = 2 + 4$ sum.

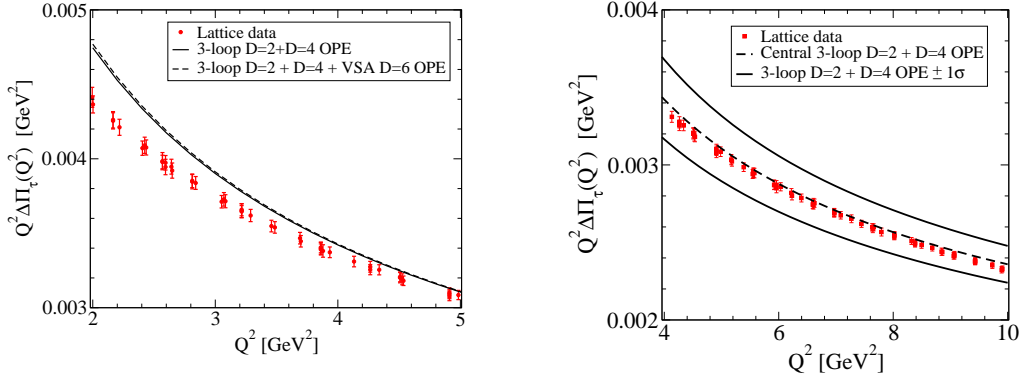


FIG. 3: Left panel: Comparison of lower- Q^2 lattice results to $D = 2 + 4$ and $D = 2 + 4 + 6$ OPE expectations (fixed-scale, 3-loop truncation for $D = 2$, VSA for $D = 6$). Right panel: Lattice results and the $D = 2 + 4$ OPE sum at larger Q^2 , with conventional OPE error estimates (fixed-scale, 3-loop-truncated $D = 2$).

III. AN ALTERNATE IMPLEMENTATION OF THE FB FESR APPROACH

The results of the previous section suggest an obvious alternative to the conventional implementation of the FB FESR approach. First, the 3-loop-truncated FOPT treatment favored by the comparison to the high- Q^2 lattice results is employed for the $D = 2$ OPE integrals. Second, since both lattice and continuum results suggest that conventional implementation assumptions for the effective $D > 4$ condensates, C_D , are unreliable, we avoid such assumptions and instead fit the C_D to data. FESRs based on the weights $w_N(y)$ are particularly convenient for use in fitting the $C_{D>4}$ since the w_N -weighted OPE

integral involves only a single $D > 4$ contribution, $(-1)^N C_{2N+2}/[(N-1)s_0^N]$. The s_0 dependence of the w_N -weighted spectral integrals in the region above $s_0 \sim 2 \text{ GeV}^2$, where residual duality violations remain small, then provides sufficient information to allow both unknowns, $|V_{us}|$ and C_{2N+2} , entering the w_N FESR to be determined.

The OPE and spectral distribution inputs employed in our analysis were outlined above. The single-weight w_2 , w_3 and w_4 FESR $|V_{us}|$ fit results, $0.2226(22)_{exp}(4)_{th}$, $0.2229(22)_{exp}(4)_{th}$ and $0.2230(22)_{exp}(4)_{th}$, respectively, show a dramatically reduced weight dependence relative to those of the conventional implementation. Table I shows the error budgets for these fits. Theory errors, resulting from uncertainties in the input parameters α_s , m_s and $\langle m_s \bar{s}s \rangle$, and the small $J = 0$ continuum subtraction, are labelled by $\delta\alpha_s$, δm_s , $\delta\langle m_s \bar{s}s \rangle$ and $\delta(J = 0 \text{ sub})$, respectively, and shown above the horizontal line. Those induced by the covariances of the non-strange and strange experimental distributions $dR_{V+A;ud}/ds$ and $dR_{V+A;us}/ds$ are denoted δ_{ud}^{exp} and δ_{us}^{exp} and shown below the horizontal line. The δ_{us}^{exp} uncertainties strongly dominate the total errors.

TABLE I: Single-weight fit $|V_{us}|$ error contributions for the w_2 , w_3 and w_4 FESRs, using 3-loop-truncated FOPT for the $D = 2$ OPE series. Notation as described in the text.

Source	$\delta V_{us} $ w_2 FESR	$\delta V_{us} $ w_3 FESR	$\delta V_{us} $ w_4 FESR
$\delta\alpha_s$	0.00002	0.00006	0.00006
$\delta m_s(2 \text{ GeV})$	0.00008	0.00009	0.00008
$\delta\langle m_s \bar{s}s \rangle$	0.00035	0.00035	0.00035
$\delta(J = 0 \text{ sub})$	0.00009	0.00009	0.00009
δ_{ud}^{exp}	0.00027	0.00028	0.00028
δ_{us}^{exp}	0.00221	0.00221	0.00222

From the lattice-OPE comparison discussed above, the estimates in the upper half of the table should provide a very conservative assessment of theoretical uncertainties. Combining the different components in quadrature yields a total theory error of 0.0004 on $|V_{us}|$ for all three determinations. The new implementation of the FB FESR approach is thus competitive with the alternate $K_{\ell 3}$ and $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ determinations from a theory error point of view, though improvements to the errors on the strange experimental distributions are required to make it fully competitive over all.

To test whether fitting the $D > 4$ condensates has solved the problem of the s_0 -instabilities found in the conventional implementation, we have rerun the s_0 -dependent w_N analyses, using the central fitted C_{2N+2} values as input. The dashed lines in the right panel of Fig. 1 show the results of this exercise. Using the fitted $C_{D>4}$ values dramatically reduces the s_0 -instabilities of the conventional implementation versions of the same analyses, providing a strong self-consistency check on the new FB FESR implementation.

Given the excellent consistency of the $|V_{us}|$ results obtained from the individual w_2 , w_3 and w_4 FESRs, we quote as our final result the value obtained from a combined 3-weight fit. With our central choice of input for the us distribution, employing the preliminary BaBar result for $B[\tau^- \rightarrow K^- \pi^0 \nu_\tau]$ [13], we find

$$|V_{us}| = 0.2229(22)_{exp}(4)_{th}, \quad (10)$$

in excellent agreement with the result from $K_{\ell 3}$ and compatible within errors with the expectations of 3-family unitarity⁴.

IV. CONCLUSIONS

We have revisited the determination of $|V_{us}|$ from flavor-breaking finite-energy sum rule analyses of experimental inclusive non-strange and strange hadronic τ decay distributions, identifying an important systematic problem in the conventional implementation of this approach, and developing an alternate implementation which cures this problem. We have also used lattice results to bring under better theoretical control the treatment of the potentially problematic $D = 2$ OPE series entering these analyses. The new implementation requires fitting effective $D > 4$ condensates to data and dramatically reduces the w - and s_0 -instabilities found when conventional implementation assumptions are used. The new implementation produces results for $|V_{us}|$ in excellent agreement with that obtained from $K_{\ell 3}$, and compatible within errors with the expectations of three-family unitarity, thus resolving the long-standing puzzle of the $> 3\sigma$ low values of $|V_{us}|$ obtained from the conventional implementation of the FB FESR τ approach.

The FB FESR approach to the determination of $|V_{us}|$ has been shown to have very favorable theory errors. The limitations, at present, are entirely experimental in nature, with errors strongly dominated by those on the weighted inclusive strange spectral integrals. In this regard, it is worth noting that the errors on the lower-multiplicity *exclusive-mode* $K^- \pi^0$, $\bar{K}^0 \pi^-$, $K^- \pi^+ \pi^-$ and $\bar{K}^0 \pi^- \pi^0$ contributions, all of which are based on the much higher statistics BaBar and Belle distribution data are, at present, strongly dominated by the uncertainties on the corresponding branching fractions (which normalize the unit-normalized experimental distributions). Significant improvements to the overall error can thus be achieved through improvements to the strange exclusive-mode branching fractions without requiring simultaneous, experimentally more difficult, improvements to the associated differential distributions.

⁴ The result is reduced to $|V_{us}| = 0.2204(23)_{exp}(4)_{th}$ if the alternate 2014 HFAG [19] $B[\tau^- \rightarrow K^- \pi^0 \nu_\tau]$ normalization is used instead for the $K^- \pi^0$ distribution. Finalizing the preliminary BaBar analysis of this branching fraction is thus highly desirable. The qualitative conclusions of the analysis are, however, independent of which of these two normalizations is chosen: in both cases the conventional implementation produces $|V_{us}|$ with strong unphysical s_0 - and w -dependences, both of which are cured once the alternate implementation, in which the $C_{D>4}$ are fit to data, is employed.

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